Power Based Sizing Method for Aircraft Consuming Unconventional Energy

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Traditionally, most fixed wing and rotary wing aircraft have been powered by internal combustion engines that consume hydrocarbon fuels. Only in a few exceptional designs, such as solar powered air-vehicles, are attempts made to apply alternate energy sources. In the past decade, however, the aerospace community has shown a renewed interest in alternate energy sources for revolutionary propulsion systems. In particular, fuel cells are increasingly being considered as an alternate power source for their potential outstanding advantages over the traditional power system. Nevertheless, traditional aircraft sizing methods are not immediately applicable for such unconventional-energy consuming air-vehicle designs. This paper proposes a generalized aircraft sizing formulation that is also applicable to revolutionary aircraft concepts powered by unconventional energy sources and/or have revolutionary propulsion systems. A power based formulation, which allows easy tracking of energy transformation process from the first power generation to the last propulsive power production, is introduced. Lastly, a generalized aircraft weight estimation formulation that is also valid for unconventional-energy consuming propulsion systems is developed.

Nomenclature

C_D	=	Coefficient of clean configuration drag
C_{D_o}	=	Zero lift drag coefficient
C_{DR}	=	Coefficient of additional drag
D	=	Basic configuration drag
E	=	Amount of onboard energy
E_{CE}	=	Amount of consumable onboard energy
E_{NE}	=	Amount of non-consumable onboard energy
g_o	=	Gravity constant
h	=	Altitude
K_1	=	Drag polar coefficient for 2nd order term
K_2	=	Drag polar coefficient for 1st order term
m	=	Number of total mission segments
n	=	Load factor

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n_T = Number of total power paths
n_{CE} = Number of consumable power paths
n_{NE} = Number of non-consumable power paths
n_{NE} = Number of non-consumable power paths
n_{PD} = Number of power devices
P = Power available
P_i = Power available of i^th power path
P_{io} = Prototype power of i^th power path
P_{ioSL} = Prototype power of i^th power path at S.L.
q = Dynamic pressure
R = Additional drag to D
S_G = Ground roll
T = Thrust
u = Drag to thrust ratio
V = Free stream velocity
V_{stall} = Stall speed
V_{TO} = Take-off speed
W = Instantaneous aircraft weight
W_{CE} = Consumable energy weight
W_{Energy} = Total stored energy weight
W_E = Empty weight
W_{Energy} = Total stored energy weight
W_{NE} = Non-consumable energy weight
W_P = Payload weight
W_{TO} = Take-off gross weight
z_e = Energy height
α_i = Power lapse ratio
β = Weight fraction
Δ = Weight correction factor
Γ' = Empty weight fraction
Φ = Power devices weight fraction
ν = Stored product to fuel ratio
Π_{AE} = Consumable energy weight fraction $(dW \neq 0)$
Π_{η_i} = Overall efficiency of i^th power path
ρ = Free-stream air density
ρ_{CE_j} = Energy density of consumable energy
ρ_{NE_j} = Energy density of non-consumable energy
ρ_p = Power density of power devices
Σ_{CE} = Consumable energy weight fraction $(dW = 0)$
Σ_{NE} = Non-consumable energy weight fraction
τ_i = Power fraction of i^th power path

I. Introduction

SINCE the Wright brothers' first flight, most aircraft have been powered by internal combustion engines that consume hydrocarbon fuels. Only a few attempts have reminded the aerospace community of the obvious but not always apparent fact that aircraft can be powered by different energy sources or different power generation devices.

In recent years, several unconventional energy sources and revolutionary propulsion systems, as illustrated in Figure 1 are obtaining increased attention as alternative energy and power sources. Furthermore, development of revolutionary aerospace concepts that are not feasible with conventional propulsion systems, such as the AeroVironment Helios and vehicles designed for planetary exploration, have also provided an outlet for revolutionary propulsion technologies. Nevertheless, it is the authors' observation that traditional aircraft sizing methods are not immediately applicable to such unconventional energy consuming air-vehicles and are in need of refinement.

For instance, the most widely used aircraft weight estimation technique for conventional aircraft is based on the assumption that the rate of an aircraft's change in weight equals the fuel consumption. Aircraft that are equipped with regenerative power systems, such as the Helios, however, maintain the same weight during the entire mission. Furthermore, it is possible that more stringent emission regulations of the future may force aerospace engineers innovate propulsion systems to separate specific by-product components from engine emissions and store them onboard during flight. Such an air-vehicle should be sized differently from conventionally powered vehicles, whose combustion by-products are continuously expelled.

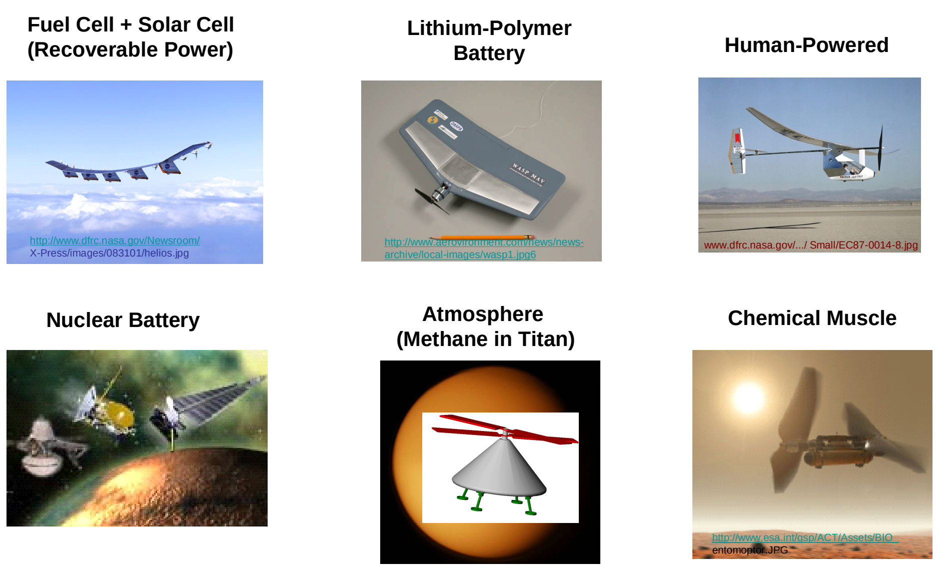

Figure 1: Unconventional Energy Sources for Air/Space-vehicles

II. Review of Traditional Aircraft Sizing and Synthesis

The goal of aircraft sizing and synthesis is not to yield an optimum aircraft design. Rather, aircraft sizing is the process that determines the *scale* of a certain configuration to satisfy all of its mission and performance requirements. Therefore, the sizing process is initiated with a notional air-vehicle configuration as depicted in Figure 1. This initial sketch is developed largely based on engineering intuition and prior knowledge and need not include every single detail about the aircraft, but it should be specific enough to establish the basic aerodynamic characteristics of the configuration (i.e. drag polar and the maximum lift coefficient). There exists a variety of first-order estimation of aerodynamic characteristics based on fundamental geometric information such as aspect ratio, taper ratio, wing thickness, etc.

Another important set of inputs for aircraft sizing is the propulsion system data, such as the thrust and specific fuel consumption (SFC) variation with altitude, Mach number and power settings. Most design projects begin with a rubberized engine which can be scaled up or down so as to match the thrust required by the mission. Therefore, the thrust lapse behavior through flight conditions and power settings, other than the figure of thrust available itself, is precisely what is required.

The traditional aircraft sizing and synthesis process consists of two primary parts: constraint analysis and mission analysis. The constraint analysis is the process to establish thrust or power matching so that the aircraft satisfies all known point performance requirements. Most aerodynamic point performance requirements which include take-off field length, climb, acceleration, sustained turn, approach speed can be expressed as functions of thrust loading (T/W) at sea level and wing loading (W/S) for a given aircraft geometry, thrust lapse behavior and flight conditions. Thus the feasible solution region can be identified by a set of constraint curves that represent each of performance constraint. In general, the best values for these two parameters can be obtained where the lowest thrust loading is located since any redundant thrust will most likely increase the propulsion system weight and take-off gross weight as a consequence. This analysis is said to determine the balance of thrust. As a complement to this, mission analysis is required to estimate the amount of fuel or stored energy to perform the entire mission. This analysis determines the required fuel fraction by calculating the product of all the weight fractions for the entire mission. In this sense, this analysis is said to determine the fuel balance. From this information and a regressed equation for the empty weight, the aircraft's take-off gross weight is obtained. Once the take-off gross weight, as well as the T/W and W/S values, are available after the two analyses, finally the available thrust and wing area of the notional configuration can be found.

The overall process of the traditional aircraft sizing method briefly outlined above can be made applicable to revolutionary aerospace concepts that consume unconventional types of energy. In order to do so, a great deal of modification in the formulation is required.

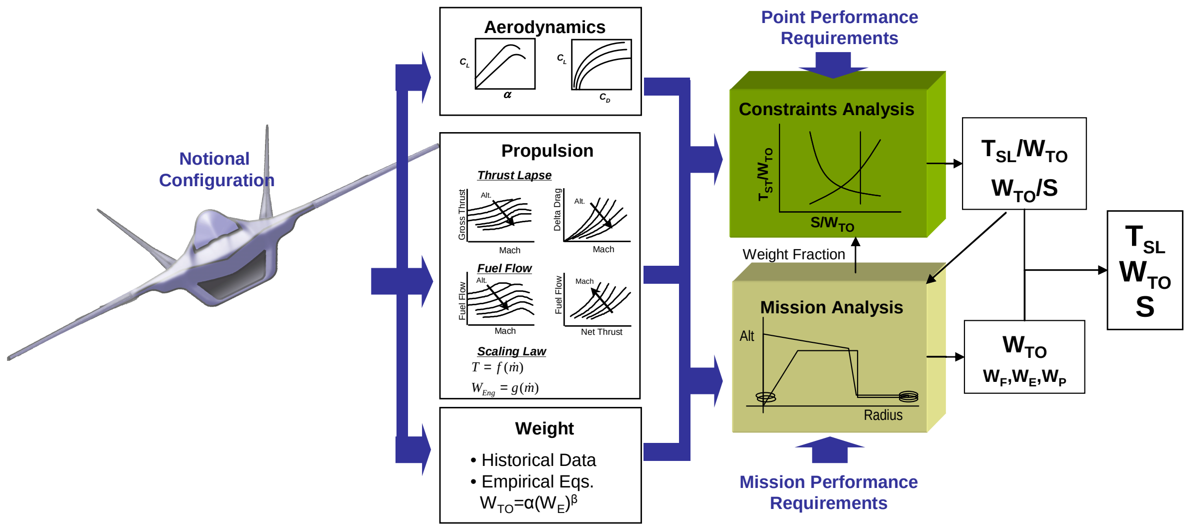

Figure 2: Overall Process of Aircraft Sizing

III. Fundamentals of New Formulation

A. Generalized Propulsion System

A propulsion system is the device that produces the propulsive thrust through a series of energy conversions. This process is typically affected by the system's design parameters and operating conditions such as Mach number, altitude and ambient temperature. Because the technology behind the traditional air-breathing combustion engines is presently matured, their thrust lapses and SFC behaviors of traditional air-breathing combustion engine are well understood. Thus, aircraft design engineers need not track every single detail of the internal energy conversion process inside the engine. Instead, designers can pick the most suitable engine type and cycle depending on aircraft's mission requirements, and consequently the thrust lapse, SFC behavior, and the scaling laws of the engine are known.

Nevertheless, emerging revolutionary propulsion systems are not yet fully developed and are continuously evolving, and thus their thrust and fuel consumption behavior, and the scaling laws are not well established. In addition, several emerging energy sources introduce more ambiguous boundaries between airframe design and propulsion system design. For example, the selection and sizing process of a fuel cell system is a significant multidisciplinary challenge in itself. Therefore, a power based formulation is desired since it allows easy tracking of the energy transformation process from the first power generation to the last propulsive power production.

The proposed formulation is initiated by defining generalized power systems. The vehicle is powered by *n* different power paths and each of them consists of multiple energy sources and energy transformation devices (E.T.D.) as shown in Figure 3. Then, the power available to the aircraft is the summation of all onboard powers.

$$P = \sum_{i=1}^{n_T} p_i \tag{1}$$

This expression can be modified by introducing a power fraction factor, τ_i.

$$P_i = \tau_i P \tag{2}$$

The first power (prototype power), P_{i_o}, generated from the original energy source is eventually converted to the final propulsive power through multiple energy transformation processes and the final output power is the product of the efficiencies and P_{i_o}, since there are losses associated with each transformation/conversion.

$$P_i = \Pi_{\eta_i} P_{i_o} \tag{3}$$

Since the prototype power may vary depending on flight conditions, it can be written in terms of the fixed sea level prototype power, $P_{i_{oSL}}$ by introducing the power lapse ratio, α_i.

$$P_{i_o} = P_{i_{oSL}} \alpha_i \tag{4}$$

Lastly, combining equation (2), (3) and (4),

$$P_{i_{oSL}} = \frac{\tau_i P}{\Pi_{\eta_i} \alpha_i} \tag{5}$$

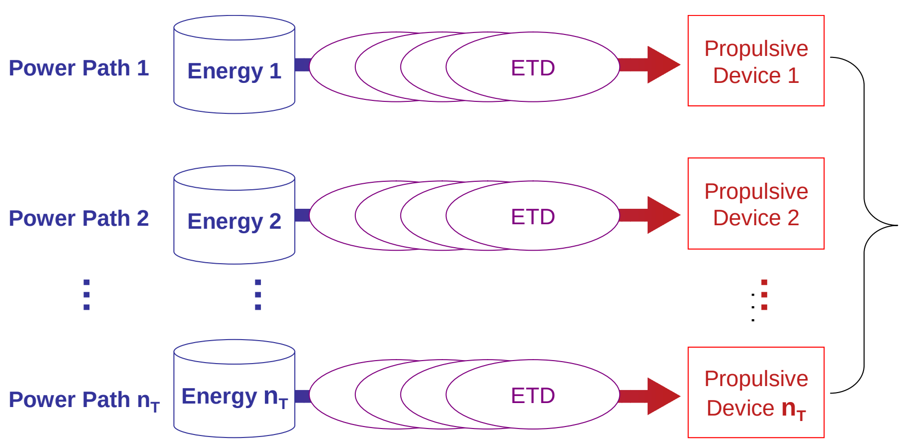

Figure 3: Power Paths from Multiple Energy Sources

B. Consideration of Multiple Energy Sources

An aircraft must store energy onboard in order to fly its mission. The conventional stored energy is in the form of hydrocarbon fuel of which weight is consumable during flight. Because, there are energy storage devices such as electric battery and perhaps that of nuclear power, whose weight stays virtually constant during their operations, it is imperative to distinguish this characteristic of each energy source. In this study, consumable energy is defined as energy that is derived from a source whose weight is reduced during power generation; i.e. traditional hydrocarbon fuels. Alternatively non-consumable energy is defined here as energy derived from a source whose weight stays constant or negligibly changes during power generation, i.e. electric battery, human power, nuclear battery.

In order to obtain a generalized formulation, it is assumed that the aircraft has multiple power paths which have either consumable or non-consumable energy sources. If the number of power paths of consumable energy is n_{CE} and the number of power paths of non-consumable energy is n_{NE}, then, the total energy stored onboard, E, is the sum of these two type of energy.

$$E = \sum_{i=1}^{n_{CE}} E_{CE_i} + \sum_{j=1}^{n_{NE}} E_{NE_j} \tag{6}$$

$$dE = \sum_{i=1}^{n_{CE}} dE_{CE_i} + \sum_{i=1}^{n_{NE}} dE_{NE_j} \tag{7}$$

Therefore, the total onboard stored energy weight W_{Energy} is

$$W_{Energy} = W_{CE} + W_{NE} = \sum_{i=1}^{n_{CE}} W_{CE_i} + \sum_{j=1}^{n_{NE}} W_{NE_j} \tag{8}$$

It is must be noted that this formulation calculates the energy weights in terms of power paths and not energy types. For instance, if JP-8 is used for both a conventional jet engine and a fuel cell system which powers the electric motor and propeller, then this system has two power paths and the required fuel weight for each of the two power paths is separately estimated.

C. Weight Differential Equation

A more generalized weight decomposition can be expressed as follows,

$$W = W_E + W_P + W_{CE} + W_R \tag{9}$$

where W_R is the weight of the retained products. It is important to realize that the non-consumable energy weight is included in the empty weight. Thus, the change in an aircraft's weight can be expressed as

$$dW = dW_{CE} + dW_R \tag{10}$$

By assuming retained products weights are proportional to fuel consumption,

$$dW_R = -\nu dW_{CE} \tag{11}$$

where, ν is stored product to fuel ratio. Substituting Eqs. (11) into Eqs. (10) yields,

$$dW = (1-\nu) dW_{CE} \tag{12}$$

Lastly by introducing a constant k,

$$dW = k dW_{CE} \tag{13}$$

where,

$$k = 1 - \nu \tag{14}$$

IV. Formulation – Part 1: Constraint Analysis

Application of Newton's second law to describe an aircraft motion yields[1]

$$\frac{P}{W_{TO}} = \beta\left\{\frac{qS}{W_{TO}}\left[K_1\left(\frac{n\beta}{q}\frac{W_{TO}}{S}\right)^2 + K_2\left(\frac{n\beta}{q}\frac{W_{TO}}{S}\right) + C_{D_o} + \frac{R}{qS}\right] + \frac{1}{V}\frac{d}{dt}\left(h + \frac{V^2}{2g_o}\right)\right\}V \tag{15}$$

where n represents the load factor. When Eqs. (5) is substituted into Eqs.(15) to signify n_T different power paths, then we finally arrive at:

$$\frac{P_{ioSL}}{W_{TO}} = \frac{\tau_i\beta}{\Pi_{\eta_i}\alpha_i}\left\{\frac{qS}{W_{TO}}\left[K_1\left(\frac{n\beta}{q}\frac{W_{TO}}{S}\right)^2 + K_2\left(\frac{n\beta}{q}\frac{W_{TO}}{S}\right) + C_{D_o} + \frac{R}{qS}\right] + \frac{1}{V}\frac{d}{dt}\left(h + \frac{V^2}{2g_o}\right)\right\}V \tag{16}$$

From Eqs (16), specific constraint equations for various performance requirements can be derived. as listed in Table I.

Table I: Constraint Equations

	Assumption	Constraint Equation
Take-off	$n=1$, $\frac{dh}{dt}=0$, $V_{TO}=k_{TO}V_{STALL}$, $P >> (D+R)V$	$\frac{P_{ioSL}}{W_{TO}} = \frac{2}{3}\frac{\beta^2\tau_i k_{TO}^2}{\Pi_{\eta_i}\alpha_i S_G \rho g_o}\left(\frac{W_{TO}}{S}\right)^{\frac{3}{2}}$
Constant Speed Climb	$\frac{dV}{dt}=0$, $n\approx 1$	$\frac{P_{ioSL}}{W_{TO}} = \frac{\tau_i\beta V}{\Pi_{\eta_i}\alpha_i}\left\{K_1\frac{\beta}{q}\left(\frac{W_{TO}}{S}\right) + K_2 + \frac{C_{D_o}+C_{DR}}{\frac{\beta}{q}\left(\frac{W_{TO}}{S}\right)} + \frac{1}{V}\frac{dh}{dt}\right\}$
Cruise	$\frac{dh}{dt}=0$, $\frac{dV}{dt}=0$, $n=1$	$\frac{P_{ioSL}}{W_{TO}} = \frac{\tau_i\beta V}{\Pi_{\eta_i}\alpha_i}\left\{K_1\frac{\beta}{q}\left(\frac{W_{TO}}{S}\right) + K_2 + \frac{C_{D_o}+C_{DR}}{\frac{\beta}{q}\left(\frac{W_{TO}}{S}\right)}\right\}$
Sustained Turn	$\frac{dh}{dt}=0$, $\frac{dV}{dt}=0$, $n=\left\{1+\left(\frac{\Omega V}{g_o}\right)^2\right\}^{\frac{1}{2}}$ or $n=\left\{1+\left(\frac{V}{g_oR_c}\right)^2\right\}^{\frac{1}{2}}$	$\frac{P_{ioSL}}{W_{TO}} = \frac{\tau_i\beta V}{\Pi_{\eta_i}\alpha_i}\left\{K_1 n^2\frac{\beta}{q}\left(\frac{W_{TO}}{S}\right) + K_2 n + \frac{C_{D_o}+C_{DR}}{\frac{\beta}{q}\left(\frac{W_{TO}}{S}\right)}\right\}$
Service Ceiling	$\frac{dh}{dt}=100$ ft/sec, $\frac{dV}{dt}=0$, $n=1$	$\frac{P_{ioSL}}{W_{TO}} = \frac{\tau_i\beta V}{\Pi_{\eta_i}\alpha_i}\left\{K_1\frac{\beta}{q}\left(\frac{W_{TO}}{S}\right) + K_2 + \frac{C_{D_o}+C_{DR}}{\frac{\beta}{q}\left(\frac{W_{TO}}{S}\right)} + \frac{1}{V}\frac{dh}{dt}\right\}$

Unlike the equations of Mattingly[1], there are a set of constraint equations for each power path. The sea level prototype power required for each power path is the total required power multiplied by the power fraction factor, $\frac{\tau_i}{\Pi_{\eta_i}\alpha_i}$. Thus, it would be easier to construct each constraint curve using Eqs (15) and then apply Eqs (5) to obtain the constraint curves for each power path. However, it should be noted that the power fraction factor varies through flight regime and thus the sea level power to weight ratio for each power path must be determined from all constraint curves.

The weight fraction, β value for each mission segment is not available at this moment. Therefore this value must be reasonably assumed for this analysis as described in Ref [1]. This assumed weight fraction values must be assessed with the mission analysis.

V. Formulation – Part 2: Mission Analysis

A. Consumable Energy Sizing

By introducing power contribution ratio, τ_i and τ_j

$$P_{CE_i} = \frac{\tau_i P}{\Pi(\eta_{CE_i})} \tag{17}$$

and

$$P_{NE_j} = \frac{\tau_j P}{\Pi(\eta_{NE_j})} \tag{18}$$

where, $\sum_{i=1}^{n_{CE}} \tau_i + \sum_{i=1}^{n_{NE}} \tau_j = 1$ and $\tau_i, \tau_j > 0$.

Consumed energy is power multiplied by time.

$$dE_{CE_i} = -P_{CE_i} dt = -\frac{\tau_i P}{\Pi(\eta_{CE_i})} dt \tag{19}$$

$$dE_{NE_i} = -P_{NE_j} dt = -\frac{\tau_j P}{\Pi(\eta_{NE_j})} dt \tag{20}$$

The change in the amount of the consumable energy is proportional to the change in weight of the energy sources.

$$dE_{CE_i} = \rho_{CE_i} dW_{CE_i} \tag{21}$$

$$dW_{CE_i} = -\frac{\tau_i P}{\rho_{CE_i} \Pi(\eta_{CE_i})} dt \tag{22}$$

where ρ_{CE_i} is the energy density of the fuel in the power path. Note $1/\rho_{CE}\Pi(\eta_{CE_i})$ is equivalent to the power specific fuel consumption (PSFC) of conventional combustion engines.

1. *Variable Aircraft Weight (k≠0)*

Substituting Eq. (22) into Eq. (13),

$$\frac{dW}{W} = -\sum_{i=1}^{n_{CE}} \frac{k\tau_i}{\rho_{CE_i} \Pi(\eta_{CE_i})} \left(\frac{P}{W}\right) dt \tag{23}$$

In order to calculate Eq. (23), $\left(\frac{P}{W}\right) dt$ should be evaluated.

Positive Excess Power

In the case of positive excess power,

$$\frac{P}{W}dt = \frac{d\left(h + V^2/2g_0\right)}{1-u} = \frac{dz_e}{1-u} \tag{24}$$

where

$$u = \frac{D+R}{T} = nV(\frac{C_D + C_{DR}}{C_L}) \Bigg/ \sum_{i=1}^{n_T} \frac{\Pi_{\eta_i}\alpha_i}{\beta}\frac{P_{i\,o\,SL}}{W_{TO}} \tag{25}$$

Therefore,

$$\frac{dW}{W} = -\sum_{i=1}^{n_{CE}} \frac{k\tau_i}{\rho_{CE_i}\Pi(\eta_{CE_i})}\frac{dz_e}{1-u} \tag{26}$$

By integrating Eqs (26), the weight fraction of the s^th^ mission segment is obtained as

$$\int_{W^{(s-1)}}^{W^{(s)}} \frac{dW}{W} = \sum_{i=1}^{n_{CE}} \left(\int_{z_e^{(s-1)}}^{z_e^{(s)}} \frac{-k\tau_i}{\rho_{CE_i}\Pi(\eta_{CE_i})}\frac{dz_e}{1-u} \right) \tag{27}$$

$$\frac{W^{(s)}}{W^{(s-1)}} = \exp\left(\sum_{i=1}^{n_{CE}} \left(\int_{z_e^{(s-1)}}^{z_e^{(s)}} \frac{-k\tau_i}{\rho_{CE_i}\Pi(\eta_{CE_i})}\frac{dz_e}{1-u} \right)\right) \tag{28}$$

If the segment is small enough such that $-k\tau_i \big/ \rho_{CE_i}\Pi(\eta_{CE_i})(1-u)$ can be assumed to be constant, then the above equations can be approximated as follows,

$$\frac{W^{(s)}}{W^{(s-1)}} = \exp\left(\sum_{i=1}^{n_{CE}} \left(\frac{-k\tau_i}{\rho_{CE_i}\Pi(\eta_{CE_i})}\frac{\Delta z_e}{1-u} \right)\right) \tag{29}$$

Zero Excess Power

In the case of zero excess power such as in cruise or sustained turn, the power required is equal to the total drag multiplied by the free-stream velocity. Therefore Eqs. (23) becomes

$$\frac{dW}{W} = -\sum_{i=1}^{n_{CE}} \frac{k\tau_i}{\rho_{CE_i}\Pi(\eta_{CE_i})}\left(\frac{D+R}{W}\right)Vdt \tag{30}$$

$$\frac{W^{(s)}}{W^{(s-1)}} = \exp\left(\sum_{i=1}^{n_{CE}} \left(\frac{-k\tau_i}{\rho_{CE_i}\Pi(\eta_{CE_i})}\left(\frac{D+R}{W}\right)V\Delta t \right)\right) \tag{31}$$

The weight fraction equations of Eqs (29) and (31) can be further simplified for each mission type by combing the associated assumptions. The simplified equations are summarized in Table II. Therefore, the ratio of the final aircraft weight to the initial aircraft weight is obtained as follows.

$$\Pi_{CE} = \left(\frac{W^{(1)}}{W^{(0)}}\right) \times \left(\frac{W^{(2)}}{W^{(1)}}\right) \times \ldots \times \left(\frac{W^{(s)}}{W^{(s-1)}}\right) \times \ldots \times \left(\frac{W^{(m)}}{W^{(m-1)}}\right) \tag{32}$$

where, m is the number of total mission segments.

Table II: Weight Fraction Equations

Warm up	$\frac{W^{(s)}}{W^{(s-1)}} = \exp\left(\sum_{i=1}^{n_{CE}}\left(\frac{-k\tau_i}{\rho_{CE_i}\Pi(\eta_{CE_i})}\sum_{i=1}^{n_T}\left(\frac{\Pi_{\eta_i}\alpha_i}{\beta}\frac{P_{i\,o\,SL}}{W_{TO}}\right)\Delta t\right)\right)$
Take-off Acceleration	$\frac{W^{(s)}}{W^{(s-1)}} = \exp\left(\sum_{i=1}^{n_{CE}}\left(\frac{-k\tau_i}{\rho_{CE_i}\Pi(\eta_{CE_i})}\frac{V_{TO}}{1-u}\right)\right)$
Take-off Rotation	$\frac{W^{(s)}}{W^{(s-1)}} = \exp\left(\sum_{i=1}^{n_{CE}}\left(\frac{-k\tau_i}{\rho_{CE_i}\Pi(\eta_{CE_i})}\sum_{i=1}^{n_T}\left(\frac{\Pi_{\eta_i}\alpha_i}{\beta}\frac{P_{i\,o\,SL}}{W_{TO}}\right)\Delta t_R\right)\right)$
Constant Speed Climb	$\frac{W^{(s)}}{W^{(s-1)}} = \exp\left(\sum_{i=1}^{n_{CE}}\frac{-k\tau_i}{\rho_{CE_i}\Pi(\eta_{CE_i})}\frac{\Delta h}{1-u}\right)$
Cruise	$\frac{W^{(s)}}{W^{(s-1)}} = \exp\left(\sum_{i=1}^{n_{CE}}\frac{-k\tau_i}{\rho_{CE_i}\Pi(\eta_{CE_i})}\frac{(C_D + C_{DR})\Delta s}{C_L}\right)$
Sustained Turn	$\frac{W^{(s)}}{W^{(s-1)}} = \exp\left(\sum_{i=1}^{n_{CE}}\frac{-k\tau_i}{\rho_{CE_i}\Pi(\eta_{CE_i})}\frac{(C_D + C_{DR})}{C_L / n}\frac{2\pi NV}{g_o\sqrt{n^2-1}}\right)$
Loiter	$\frac{W^{(s)}}{W^{(s-1)}} = \exp\left(\sum_{i=1}^{n_{CE}}\frac{-k\tau_i}{\rho_{CE_i}\Pi(\eta_{CE_i})}\frac{(C_D + C_{DR})V\Delta t}{C_L}\right)$

In the traditional aircraft sizing, the fuel fraction is obtained as follows

$$\frac{W_{landing}}{W_{TO}} = \frac{W_{TO} - W_F}{W_{TO}} = \Pi \tag{33}$$

or

$$\frac{W_F}{W_{TO}} = \frac{W_{TO} - W_{landing}}{W_{TO}} = 1 - \Pi \tag{34}$$

where, Π is the product of the weight fractions throughout the entire mission segments. The proposed formulation, however, accounts for the effect of retaining the by-products.

$$\frac{W_{landing}}{W_{TO}} = \frac{W_{TO} - W_{CE} + W_R}{W_{TO}} = \Pi_{CE} \tag{35}$$

where, Π_{CE} is the product of the weight fractions throughout the entire mission segments.

Combining Eqs (35) with Eqs (11) and (14) yields,

$$\frac{W_{CE}}{W_{TO}} = \frac{(1 - \Pi_{CE})}{k} \tag{36}$$

Once this analysis is completed, the assumed weight fraction β at each mission segment in the constraint analysis must be evaluated with the results from weight fraction analysis. If the assumption was not sufficiently accurate, it may be necessary to iterate the process up to this point since the remaining analyses including non-consumable energy sizing do not affect the weight fraction values.

2. *Constant Aircraft Weight (k= 0)*

When the aircraft's weight does not change during flight and the propulsion system consumes a certain type of fuel, the fuel consumption is in proportion to power consumption. By dividing equation (22) by the aircraft weight,

$$\frac{dW_{CE}}{W} = -\sum_{i=1}^{n_{CE}} \frac{\tau_i}{\rho_{CE_i}\Pi(\eta_{CE_i})}\left(\frac{P}{W}\right)dt \tag{37}$$

Note that W here is constant. By integrating equation (17) to flight time or flight distance of the s^th^ mission segment,

$$\frac{W_{CE}^{(s)}}{W} = \int_{t_s}^{t_{s-1}} \sum_{i=1}^{n_{CE}} \frac{\tau_i}{\rho_{CE_i}\Pi(\eta_{CE_i})}\left(\frac{P}{W}\right)dt \tag{38}$$

or

$$\frac{W_{CE}^{(s)}}{W} = \int_{s_s}^{s_{s-1}} \sum_{i=1}^{n_{CE}} \frac{\tau_i}{\rho_{CE_i}\Pi(\eta_{CE_i})}\left(\frac{P}{W}\right)\frac{ds}{V} \tag{39}$$

The total amount of required fuel is obtained by summing up Eqs. (18) or (19) through entire mission segments.

$$\frac{W_{CE}}{W_{TO}} = \sum_{s=1}^{m}\int_{t_s}^{t_{s-1}} \sum_{i=1}^{n_{CE}} \frac{\tau_i}{\rho_{CE_i}\Pi(\eta_{CE_i})}\left(\frac{P}{W}\right)dt \tag{40}$$

Therefore, fuel weight can be expressed as follows.

$$W_{CE} = \Sigma_{CE} W_{TO} \tag{41}$$

where,

$$\Sigma_{CE} = \sum_{s=1}^{m}\int_{t_s}^{t_{s-1}} \sum_{i=1}^{p} \frac{\tau_i}{\rho_{CE_i}\Pi(\eta_{CE_i})}\left(\frac{P}{W}\right)dt \tag{42}$$

B. Non-Consumable Energy Sizing

$$\frac{dE_{NE_j}}{\beta W_{TO}} = -\frac{\tau_j}{\Pi(\eta_{NE_j})}\frac{P}{W}dt \tag{43}$$

The amount of the j^th^ energy consumed in the s^th^ segment can be expressed as,

$$\frac{E_{NE_j}^{(s)}}{W_{TO}} - \frac{E_{NE_j}^{(s+1)}}{W_{TO}} = \int_{t_{s+1}}^{t_s} \frac{\beta\tau_j}{\Pi(\eta_{NE_j})}\frac{P}{W}dt \tag{44}$$

Total non-consumable energy can be calculated by summation of consumed energy of all segments.

$$\left(\frac{E_{NE_j}^{(0)}}{W_{TO}} - \frac{E_{NE_j}^{(1)}}{W_{TO}}\right) + \left(\frac{E_{NE_j}^{(1)}}{W_{TO}} - \frac{E_{NE_j}^{(2)}}{W_{TO}}\right) + \ldots + \left(\frac{E_{NE_j}^{(s-1)}}{W_{TO}} - \frac{E_{NE_j}^{(s)}}{W_{TO}}\right) + \ldots + \left(\frac{E_{NE_j}^{(m-1)}}{W_{TO}} - \frac{E_{NE_j}^{(m)}}{W_{TO}}\right) = \frac{E_{NE_j}^{(0)}}{W_{TO}} - \frac{E_{NE_j}^{(m)}}{W_{TO}} = \frac{E_{NE_j}}{W_{TO}} \tag{45}$$

Therefore,

$$\frac{E_{NE_j}}{W_{TO}} = \sum_{s=1}^{m}\left(\int_{t_s}^{t_{s-1}} \frac{\beta\tau_j}{\Pi(\eta_{NE_j})}\frac{P}{W}dt\right) \tag{46}$$

and

$$W_{NE} = \sum_{j=1}^{n_{NE}} W_{NE_j} = \sum_{j=1}^{n_{NE}} \frac{E_{NE_j}}{\rho_{NE_j}} \tag{47}$$

Finally, the total non-consumable energy is expressed as

$$W_{NE} = W_{TO}\Sigma_{NE} \tag{48}$$

where

$$\Sigma_{NE} = \sum_{s=1}^{m}\left(\sum_{j=1}^{n_{NE}}\left(\int_{t_s}^{t_{s-1}} \frac{\beta\tau_j}{\rho_{NE_j}\Pi(\eta_{NE_j})}\frac{P}{W}dt\right)\right) \tag{49}$$

C. Aircraft Weight Estimation

1. Variable Aircraft Weight ($k \neq 0$)

From Eqs (9)

$$W_{TO} = W_E + W_P + W_{CE} \tag{50}$$

The payload weight W_P is usually given as part of the customer requirements. The consumable energy weight is determined by Eqs (41). Traditionally, the empty weight is expressed in terms of empty weight fraction, Γ, multiplied by the take-off gross weight.

$$W_E = \Gamma W_{TO} \tag{51}$$

The empirical empty weight fraction equation (Γ) is not available for revolutionary concepts. Still, a relationship between the empty weight and the take-off gross weight is required in the proposed formulation. It must be noted that the empty weight may include non-consumable energy and weight of power devices which must be considered separately. Thus, the empty weight can be expressed as follows.

$$W_E = W_E' + W_{PD} + W_{NE} + \delta W_E \tag{52}$$

W' is the traditional empty weight to take-off gross weight ratio as function of take-off gross weight. W_{PD} is the weight of power devices and δW_E is the weight correction to the traditional empty weight. Then,

$$W_E = \Gamma' W_{TO} + \Phi W_{TO} + \Sigma_{NE} W_{TO} + \Delta W_{TO} \tag{53}$$

$$\Phi = \sum_{i=1}^{n_T}\sum_{k=1}^{n_{PD_i}} \frac{\beta_{Power\ Max}}{\rho_{p_{i,k}}}\left(\frac{P}{W}\right)_{Power\ Max} \tag{54}$$

By combining Eqs (36), (50) and (53),

$$W_{TO} = \frac{kW_P}{k(1-\Gamma'-\Delta-\Phi-\Sigma_{NE})-(1-\Pi_{CE})} \tag{55}$$

2. Constant Aircraft Weight (k= 0)

By combing Eqs (41), (50) and (53),

$$W_{TO} = (\Gamma' + \Delta + \Phi + \Sigma_{NE})W_{TO} + W_P + \Sigma_{CE} W_{TO} \tag{56}$$

or

$$W_{TO} = \frac{W_P}{1 - \Gamma' - \Delta - \Phi - \Sigma_{NE} - \Sigma_{CE}} \tag{57}$$

VI. Conclusion

This paper proposes a more generalized formulation which is also applicable for sizing aircraft consuming unconventional types of energy. The formulation is based on the generalization of the following three parts of the traditional aircraft sizing method. First, fuel is generalized as a concept of onboard energy which embraces variety of energy sources that can be categorized into consumable energy and non-consumable energy. Secondly, the propulsion system is modeled as an integration of multiple power paths, each of which is characterized with three distinguishing parameters: the energy density of the energy source, the power density and the efficiency maps of power transfer devices. Therefore, this method can capture key characteristics of any new propulsion system and incorporate those into the aircraft sizing process. Lastly, a more generalized weight decomposition and weight differential equation are employed.

As a result, the formulation converts the two hands of traditional aircraft sizing - “thrust and fuel balance” to “power and energy balance”. In fact, there is one more important balance that must be achieved - the balance between the required aircraft volume and the available aircraft volume. In general, the volume balance is verified through more detail studies of the internal arrangement after the initial aircraft configuration is fully established. In the case of traditional aircraft design, however, the volume balance is implicitly secured to certain degree via the application of historical regression rules to weight estimation. This is simply because all existing aircraft, whose weight data are used to construct the regressed equations, contain all subsystems, structures and fuel inside the aircraft. In addition, it is not too far fetched to consider the aircraft being designed to be a small perturbation from the historical trend. Nevertheless, when designing unconventional aircraft which uses unconventional propulsion system and consumes unconventional energy, such an implicit volume balance will not work. Therefore early consideration of the impact on the required volume due to unconventional energy and/or propulsion system will be of importance. The volumetric sizing method which is one of active researches of Aerospace System Design Laboratory (ASDL) at Georgia institute of Technology can be considered as a fulfillment of the sizing method for unconventional aircraft.

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VIII. Reference

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